TRAJECTORIES OF ANTIMATTER BODIES IN OUR SOLAR SYSTEM ACCORDING TO ISODUAL THEORY OF ANTIMATTER AND POSSIBLE IMPACTS WITH EARTH[1]

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Abstract

R. M. Santilli has proposed the innovative isodual theory of antimatter for the classical representation of neutral or charged antimatter bodies in a way compatible with available classical experimental evidence on antimatter. Santilli's isodual theory is equivalent to charge conjugation at the operator level, thus being compatible with experimental data on antimatter also at the particle level. A first prediction of the isodual theory holding at all levels of study, from the isodual Newtonian mechanics to the isodual Riemannian geometry, is that matter and antimatter experience a gravitational repulsion (antigravity) due to the negative value of the curvature tensor requested by matter-antimatter conjugation. A second prediction of the isodual theory is the existence of antimatter galaxies and, of course, antimatter asteroids traveling in space, that could represent a threat to our planet. Since Earth appears to have been hit in the past by antimatter asteroids (as it seems to be the case for the 1908 Tunguska explosion in Siberia), the author presents in this paper, apparently for the first time, a study of possible trajectories that would lead an antimatter asteroid to collide with our planet under the assumption of the repulsive gravitational interaction between Earth and the antimatter asteroid. Moreover, an estimate of the minimum approach speed required for an antimatter asteroid to impact with Earth is theoretically and numerically identified at different distances from our planet, along with other considerations about the trajectories.

[1] Supported in part by the R. M. Santilli Foundation

1. Introduction

Having been conceived before the discovery of antimatter, Newtonian mechanics, Galileian relativity and Einsteinian special and general relativities are not able to give a classical representation of neutral antimatter. As a consequence, matter and antimatter are commonly believed to differ only in the sign of the charge in contradiction with the annihilation process, quantization and other aspects. This occurrence created a scientific imbalance , because throughout the 20th century matter was studied at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely studied at the particle level (see monograph [1] for historical references and technical details).

In order to resolve this scientific imbalance, R. M. Santilli has proposed the innovative isodual theory of antimatter for the classical representation of neutral or charged antimatter bodies in a way compatible with available classical experimental evidence on antimatter. Santilli's isodual theory is equivalent to charge conjugation at the operator level, thus being compatible with experimental data on antimatter also at the particle level (see [1] for more details).

A second prediction of the isodual theory is the existence of antimatter galaxies that appear to be confirmed by recent astrophysical observations based on a new telescope with concave lenses [2] and preliminarily confirmed by independent scholars in ref. [3].

The existence of antimatter galaxies evidently implies the existence of antimatter supernova and, therefore, of antimatter asteroids traveling through the universe, with consequential danger for our planet (see Ref. [2] and papers quoted therein).

Since Earth appears to have been hit in the past by antimatter asteroids (as it seems to be the case for the 1908 Tunguska explosion in Siberia, see [4]), the author presents in this paper, apparently for the first time, a study of possible trajectories that would lead an antimatter asteroid to collide with our planet under the assumption of the repulsive gravitational interaction between Earth and the antimatter asteroid.

Other considerations are also made about the nature of these trajectories, the minimum velocity needed to impact Earth (since the Sun's gravity would repulse an approaching antimatter body slowing it down) and the minimum angle between the direction of arrival and the Sun at which an antimatter asteroid can impact Earth without being deviated. These aspects are investigated both theoretically and numerically, using an optimization process analogous to that used by the author in preceding work [5].

2. Modelization

To model the trajectory of an antimatter asteroid, we have to consider Newton's law of universal gravitation, adapting it to our case. For the generic n-th body of the Solar System, we can write:

$$F_n = G \cdot \frac{M_n \cdot M_{ast}^d}{r^2} = -G \cdot \frac{M_n \cdot M_{ast}}{r^2} \qquad (1)$$

where:

- F_n is the force applied to the antimatter asteroid by the gravity of the n-th body considered
- G is the gravitational constant
- M_n is the mass of the n-th body considered
- M^d_{ast} is the isodual mass of the antimatter asteroid
- r is the distance between the two bodies (considered point-like)

Hence the vectorial form:

$$\boldsymbol{F}_n = -G \frac{M_n M_{ast}^d}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) = G \frac{M_n M_{ast}}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) \qquad (2)$$

$$M_{ast} \left(\frac{d^2 \boldsymbol{r}}{dt^2} \right)_n = G \frac{M_n M_{ast}}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \cdot (\boldsymbol{r} - \boldsymbol{r}_n) \qquad (3)$$

where:

- $\boldsymbol{F}_n$ is the 3-components force vector
- $\boldsymbol{r}$ is the 3-components vector representing the distance of the asteroid from a fixed reference frame
- $\boldsymbol{r}_n$ is the 3-components vector representing the distance of the n-th body considered from a fixed reference frame

At this point, we must consider the effect of all the major bodies of the Solar System at the same time, making a summation of the forces due to every celestial body:

$$\frac{d^2 \boldsymbol{r}}{dt^2} = G \cdot \sum_{n=1}^{10} M_n \frac{\boldsymbol{r} - \boldsymbol{r}_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \qquad (4)$$

For the calculations all the planets were considered plus the Sun and Pluto.
In order to determine the state of the asteroid (position and velocity) at every time, we need then to solve a system of 6 non-linear differential equations:

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{Bmatrix} = \begin{Bmatrix} v_x \\ v_y \\ v_z \\ \sum_{n=1}^{10} m_n \frac{x - x_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \\ \sum_{n=1}^{10} m_n \frac{y - y_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \\ \sum_{n=1}^{10} m_n \frac{z - z_n}{|\boldsymbol{r} - \boldsymbol{r}_n|^3} \end{Bmatrix} \qquad (5)$$

The positions of the planets in this case are known data taken from available ephemeris, since the mass of the asteroid, and therefore its gravitational effect on the bodies of the Solar System, is considered negligible, so only the effects of planets on the asteroid are computed.

This system of equations, due to its complexity, can be solved only via numerical integration.
By defining a starting condition, in this case the initial sate of the asteroid, and the duration of the propagation, we can integrate the equations and obtain a trajectory in space.

At this point, with the choice of an initial time and relative initial state, we can propagate the trajectory forward in time and obtain a unique solution, but this approach makes it extremely difficult to find collisions with Earth, since *a priori* we have no idea where the initial conditions will lead the asteroid.

Another more useful approach is to give the program the final conditions, in terms of position and time (using of course the position of Earth at that time) and integrate backwards in time, in order to obtain the initial state at another arbitrary time. This way, we still have the 3 components of the final speed as degrees of freedom to obtain different trajectories in space.

It is correct to use this approach within the limits of model used, since in our approximation all forces are conservative, so the motion itself is reversible in time without violating physical laws. In the real world, we would have other interactions, such as those with solar wind and sunlight, but these interactions have not been included at this level, due to their assumed small entity and to still unsolved problems about the effects of matter light hitting an antimatter body (see paragraph 5).

3. Examples of trajectories

Here we present some examples of trajectories obtained with the above presented equations from arbitrary conditions. For clarity's sake only inner planets are represented (Sun, Mercury, Venus, Earth and Mars), while outer planets are considered in calculations but not represented in these graphs, to preserve readability.

In *Figure 1,* we have a comparison between the forward propagated trajectories of a matter asteroid and an antimatter asteroid, starting at the same position, with the same velocity and at the same time.
We can see that the two trajectories are completely different, since the matter asteroid turns left and passes behind the Sun, while the antimatter one is deflected to the right and escapes the Solar System without getting even close to the Sun.

In Figures *Figure* 2 toFigure 4 we have examples of forward propagated trajectories of antimatter asteroids in our Solar System, both close to and outside the ecliptic plane.

In Figures *Figure* 5 to *Figure* 8 we have examples of backward propagated trajectories of antimatter asteroids, whose ending position is fixed at Earth, while the impact velocity is arbitrarily chosen.

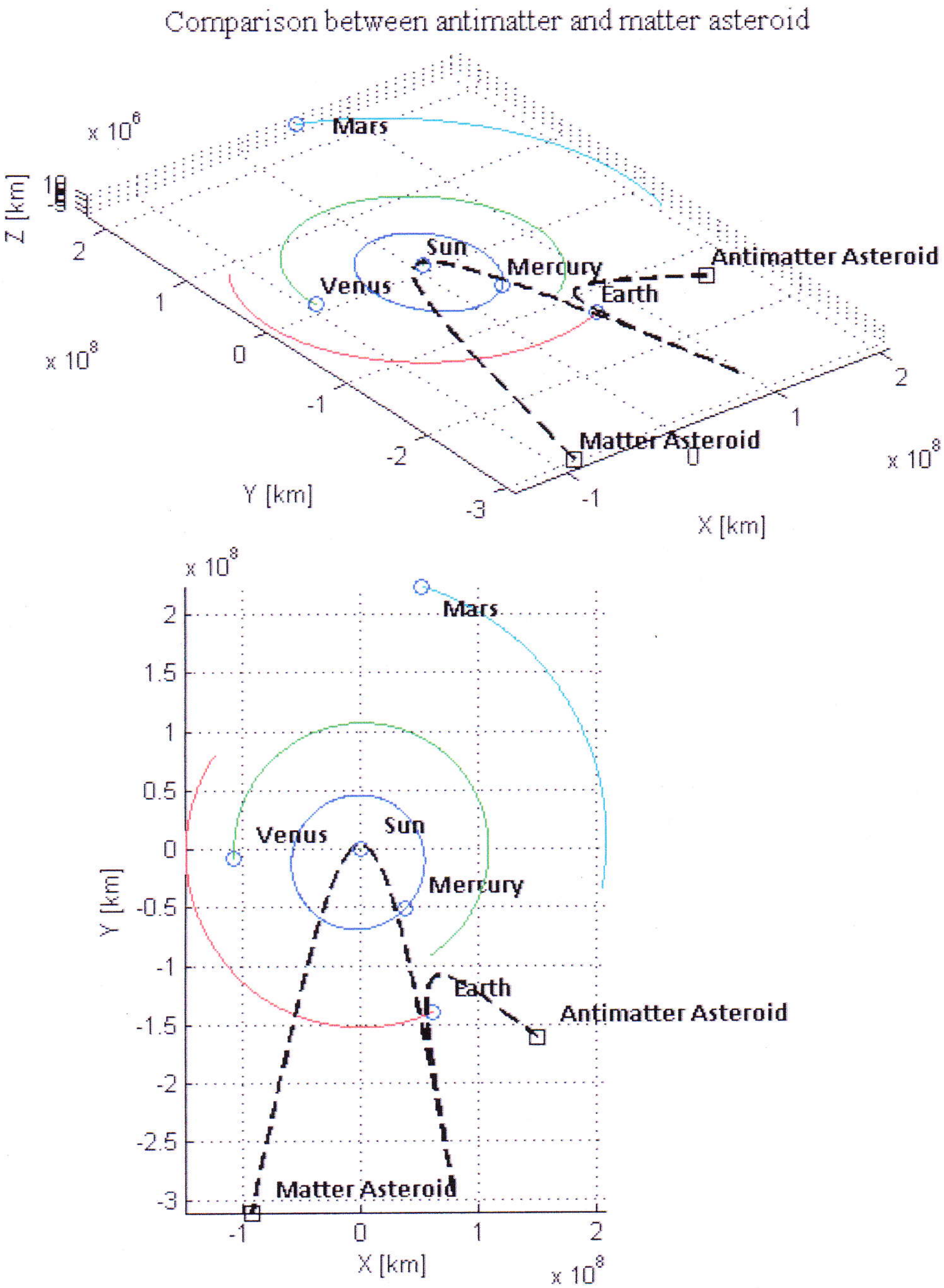

Figure 1 - A comparison between the trajectories of matter and antimatter asteroids starting from the same initial condition

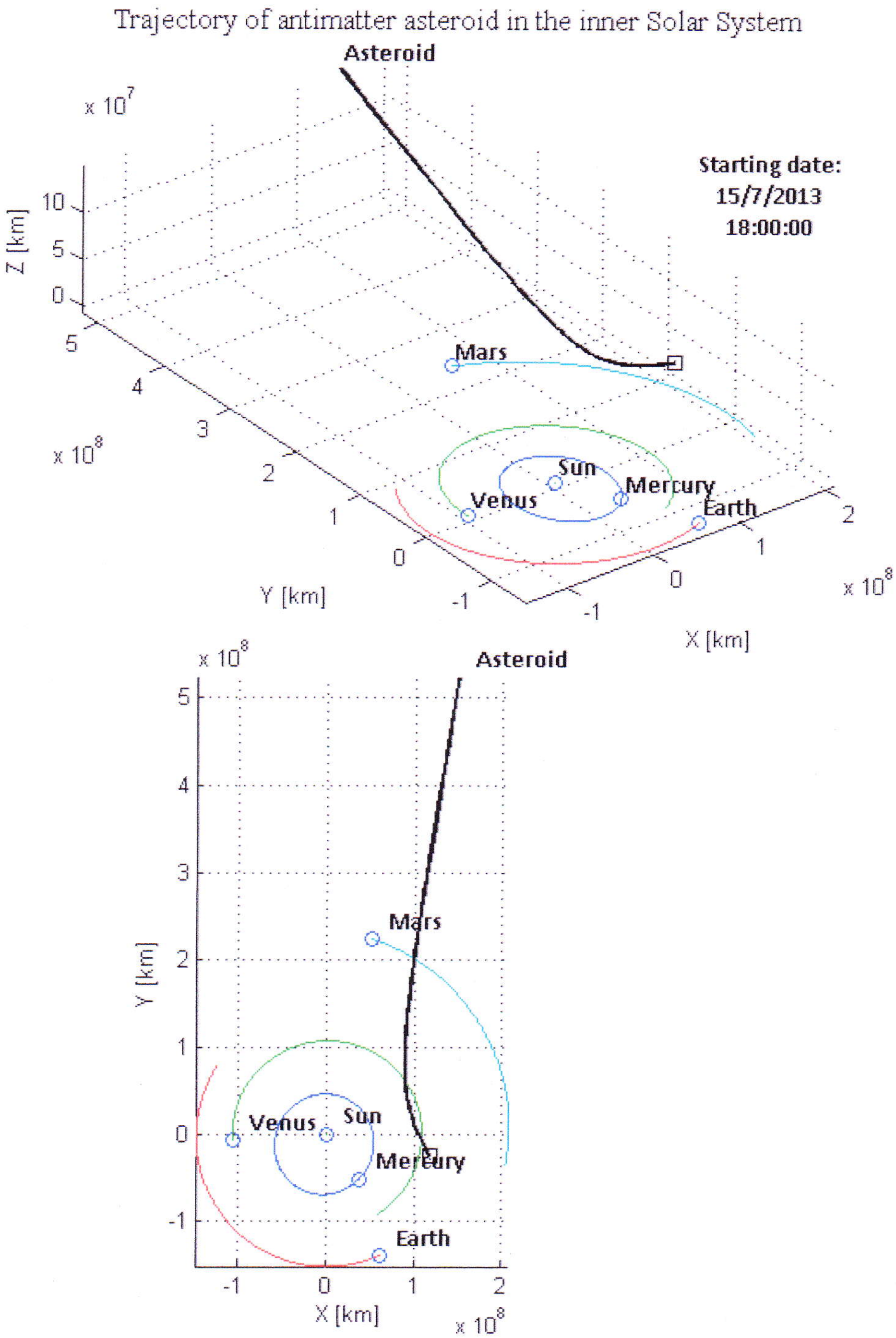

Figure 2 - Antimatter asteroid deflected before crossing the ecliptic plane

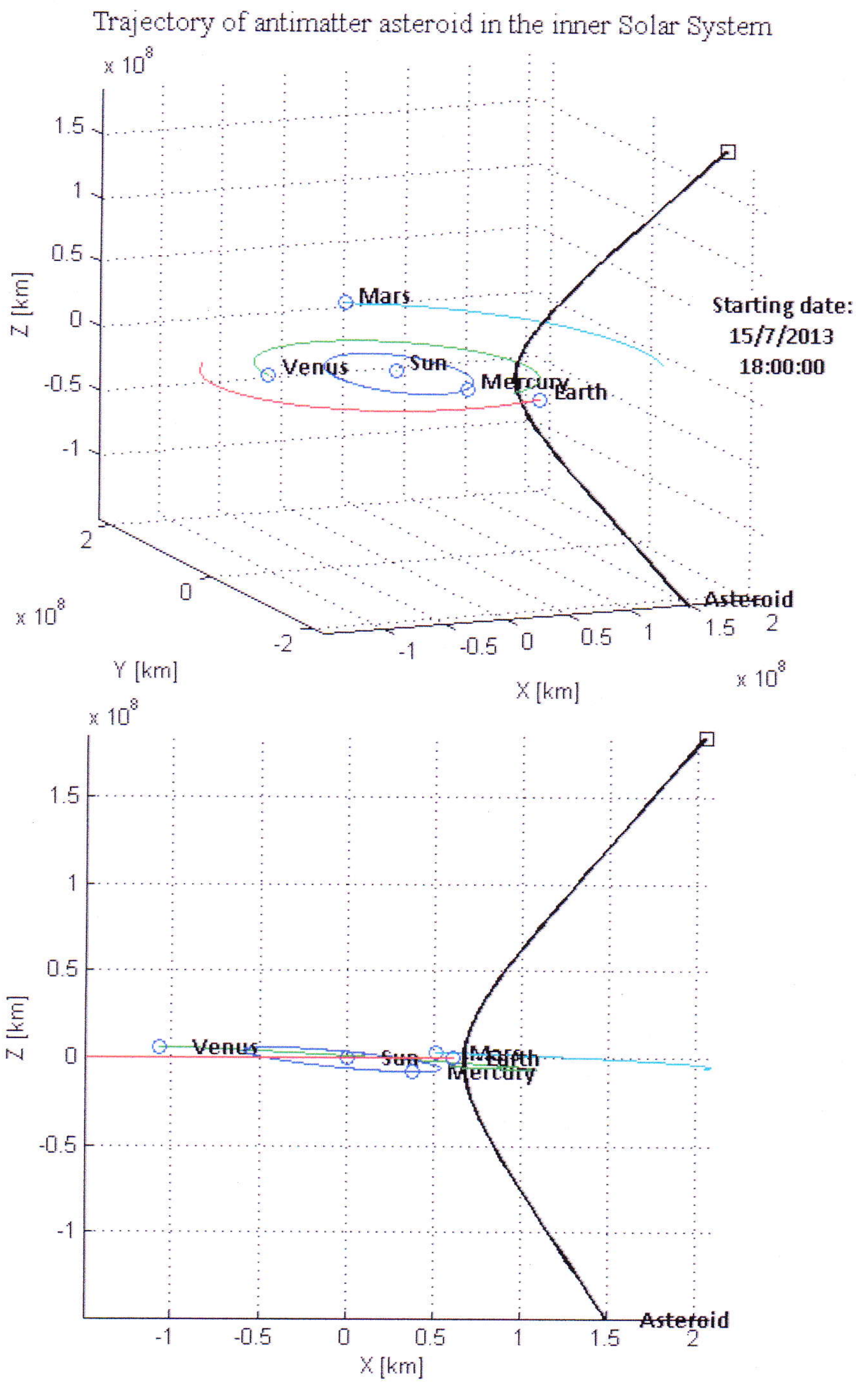

Figure 3 - Antimatter asteroid crossing the ecliptic plane almost perpendicularly

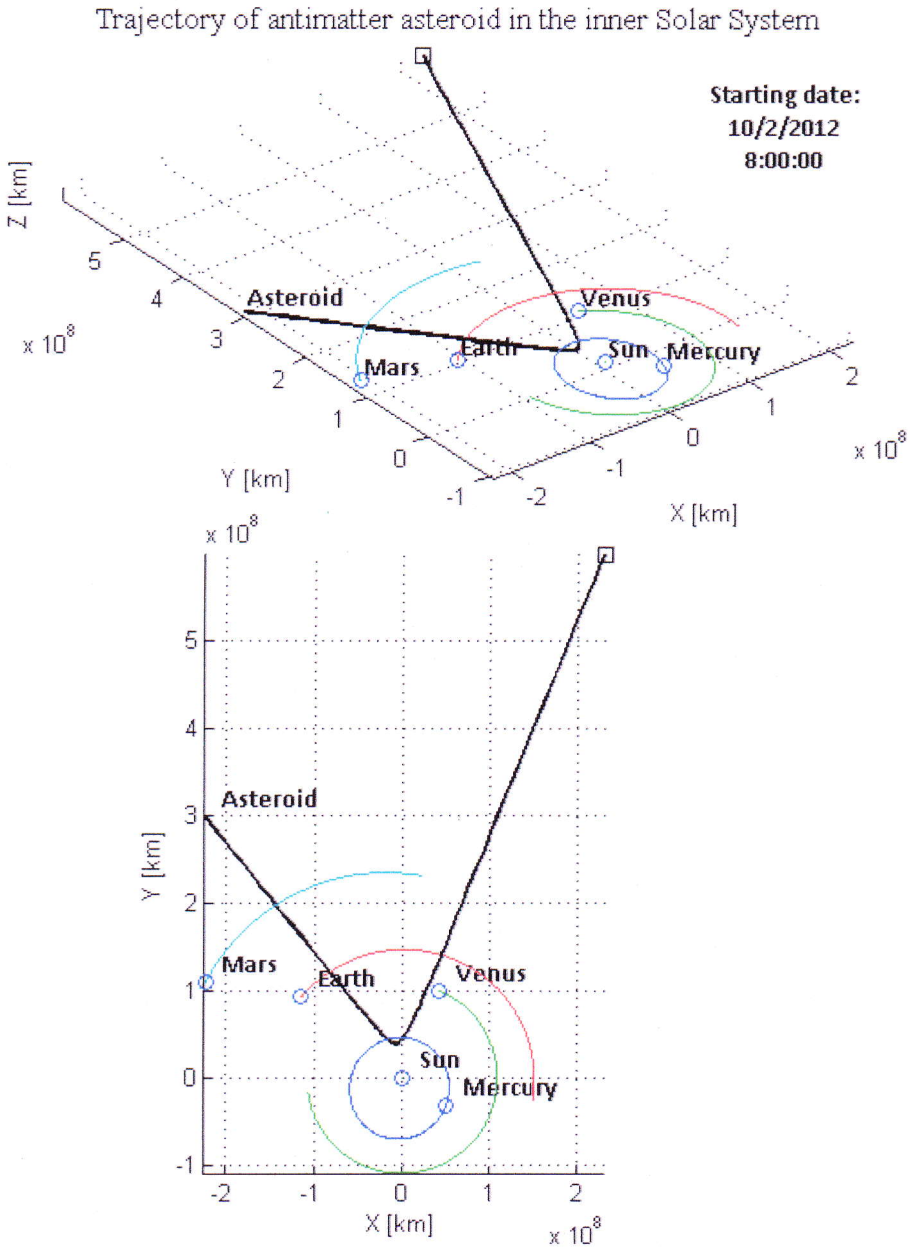

Figure 4 - Antimatter asteroid almost within ecliptic plane deflected by Sun's gravity

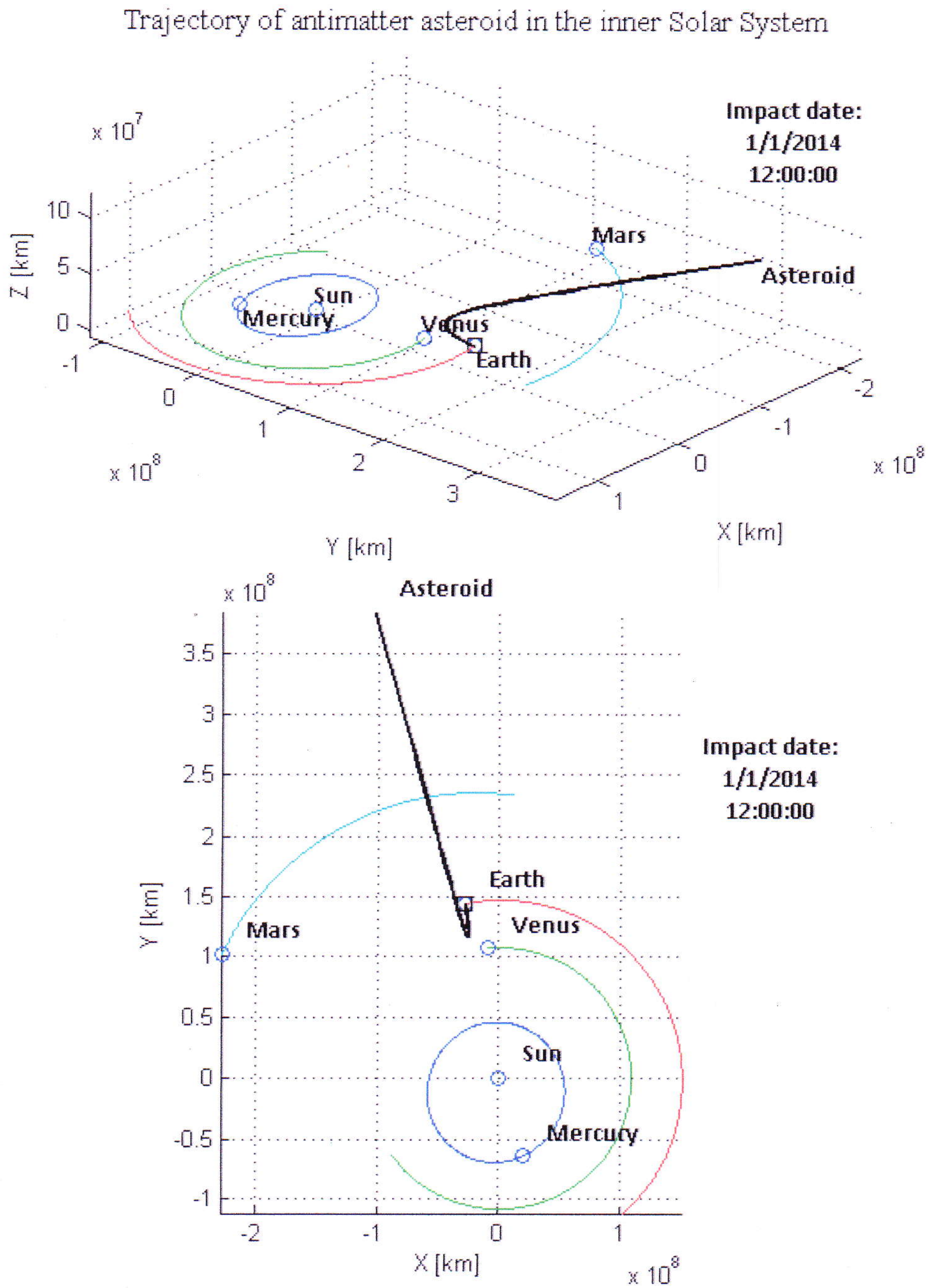

Figure 5 - Antimatter asteroid impacting Earth after having been deflected by Sun's gravity

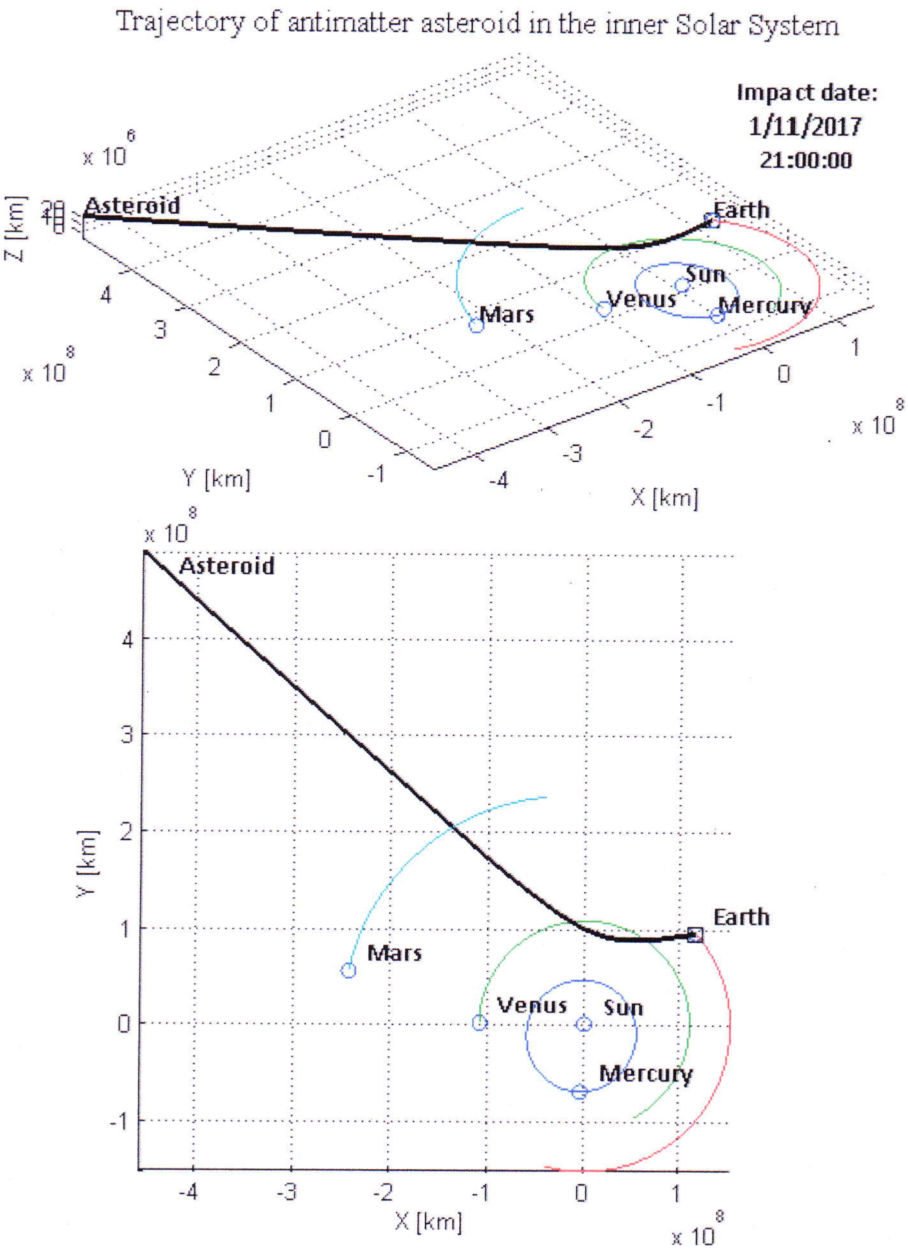

Figure 6 - Antimatter asteroid impacting Earth almost within the ecliptic plane

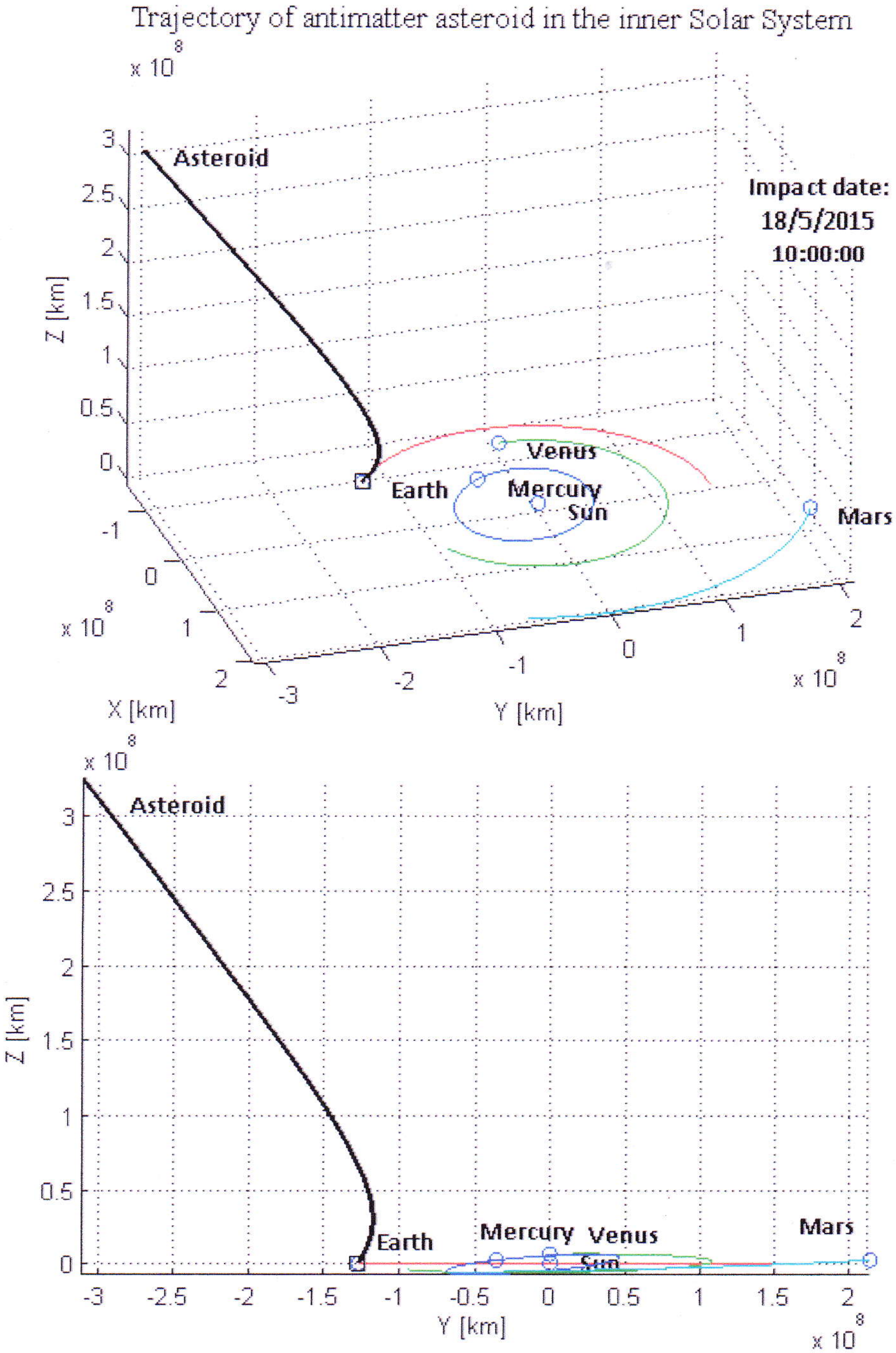

Figure 7 - Antimatter asteroid impacting Earth almost perpendicularly to the ecliptic plane

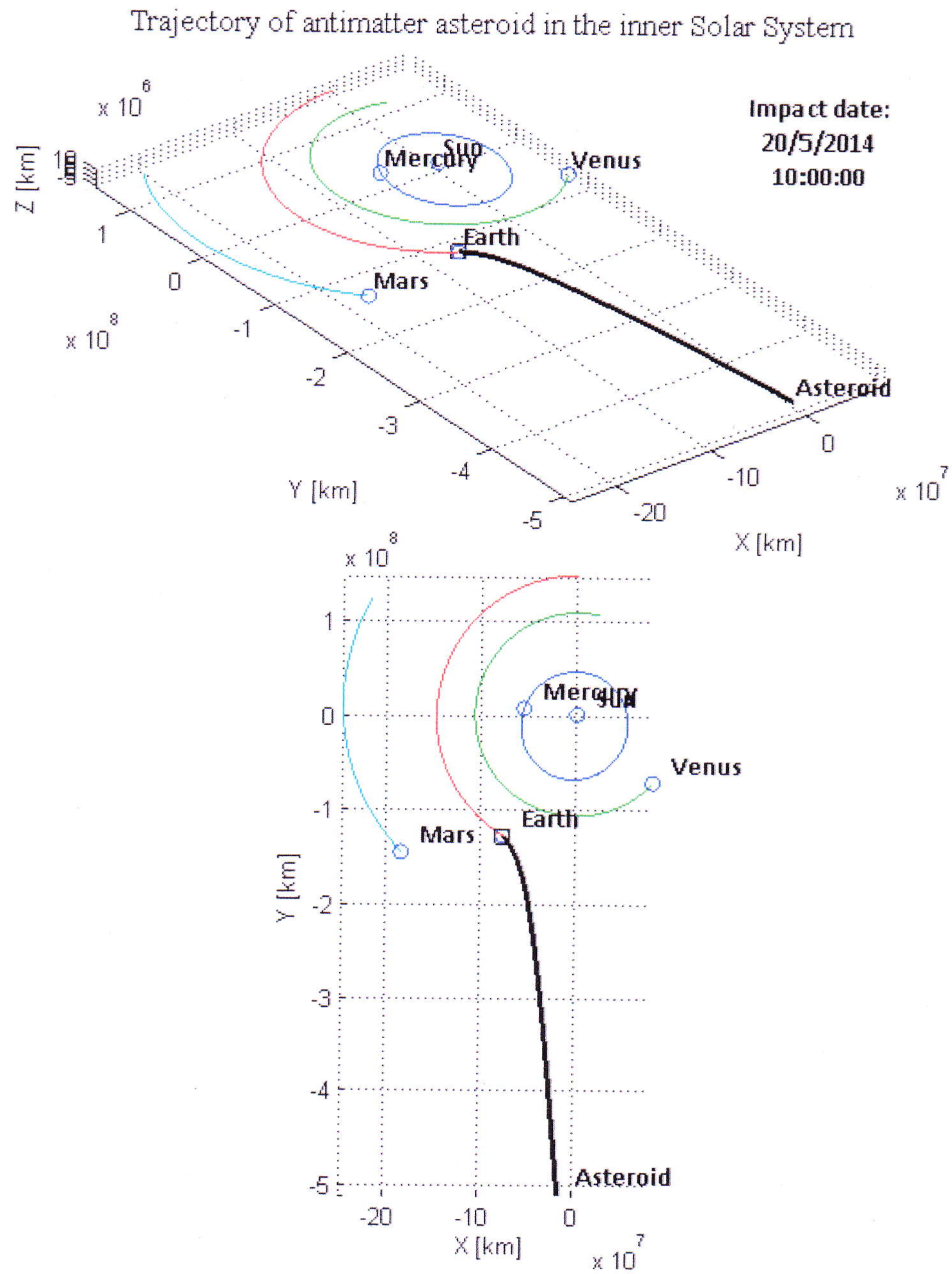

Figure 8 - Antimatter asteroid impacting Earth before being completely deflected by the Sun

4. Considerations

At this point we can make some considerations about these trajectories.
First of all, it is important to look at their shape. We can see from the pictures above that all the trajectories look very similar to hyperbolas.
It is indeed possible to verify that in the case of the two-body problem with an antimatter body and a matter "attractor", the only possible trajectory is the leg of a hyperbola located opposite to the focus with respect to the asymptotes intersection, an orbit which is impossible to obtain in the case of ordinary matter. See Ref. [6] for theoretical background.
If we consider only the Sun-Antimatter asteroid system, the conservation of energy can be written as follows:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu^d}{r} = \frac{v^2}{2} + \frac{\mu}{r} > 0 \qquad (6)$$

Where:
ε is the total energy per unit mass
v is the velocity at an arbitrary point of the trajectory
r is the distance from the center of mass of the Sun
$\mu = GM_{Sun}$ is the gravitational parameter of the Sun

From the above equation we can see that, for a generic matter-antimatter two-body system, total energy is always positive, and this implies the orbit is always open (in the case of elliptic orbits total energy is negative).
Moreover, it is possible to derive the equation of eccentricity, starting from the dynamics of the system:

$$\ddot{\boldsymbol{r}} = \frac{\mu}{r^3}\boldsymbol{r} \qquad (7)$$

Making the cross product of the two members of the equation with the vector $\boldsymbol{h}$, which represents the angular momentum, we have:

$$\ddot{\boldsymbol{r}} \times \boldsymbol{h} = \frac{\mu}{r^3}(\boldsymbol{r} \times \boldsymbol{h}) \qquad (8)$$

After some algebraic manipulations, it is possible to derive the vectorial equation of eccentricity:

$$\boldsymbol{e} = \dot{\boldsymbol{r}} \times \frac{\boldsymbol{h}}{\mu} + \hat{\boldsymbol{r}} \qquad (9)$$

Where:
$\boldsymbol{e}$ is the eccentricity vector
$\hat{\boldsymbol{r}}$ is the unit vector of the vector $\boldsymbol{r}$

From this equation we can derive, with other manipulations, the orbit equation:

$$r = \frac{h^2/\mu}{e\cos\theta - 1} \qquad (10)$$

At this point we have a representation of our trajectory in its plane, centered in the "attractor" body (the Sun in our case), in terms of radial and angular components.
Now we need to verify if it is still a conic (like in the matter case), and more precisely the kind of conic we have assumed.

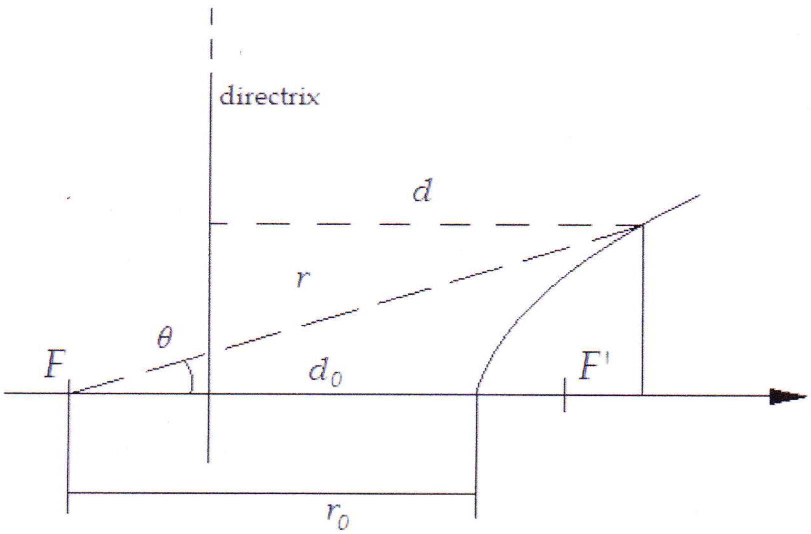

Figure 9 - Representation of a hyperbola according to the geometric definition

One definition of conic is the locus of points whose distances to some point, called a focus, and some line, called a directrix, are in a fixed ratio, called the eccentricity. Therefore we can write (see Figure 9):

$$\frac{r}{d} = \frac{r_0}{d_0} = e \qquad (11)$$

Looking at Figure 9 we can also write (note that the focus considered is the one opposite to the curve with respect to the directrix):

$$r\cos\theta - d = r_0 - d_0 \tag{12}$$

$$r(e\cos\theta - 1) = r_0(e-1)$$

At this point, with some manipulations:

$$r = \frac{r_0(e-1)}{e\cos\theta - 1} \tag{13}$$

The equation obtained is in the same form of (10), and considering (9) we can calculate the modulus of eccentricity, verifying it is always bigger than 1 (which corresponds to the hyperbolic case):

$$|\boldsymbol{e}| = \sqrt{\left(\dot{r}\frac{h}{\mu}\right)^2 + \left(\frac{h^2}{\mu r} + 1\right)^2} > 1 \tag{14}$$

With these calculations we have successfully demonstrated that the generic trajectory of a small antimatter body in the gravitational field of a massive matter body is a conic and more precisely the leg of a hyperbola opposite to the focus with respect to the axis of symmetry (or the intersection of the asymptotes).

In the trajectories above we have the presence of other planets besides the Sun, so we are definitely not in the case of the two-body problem, but still the trajectories look very much like hyperbolas. This is because the gravitational effect of the planets is very small if compared to the effect of the Sun, and negligible outside their spheres of influence.

Another element that is worth investigating is the problem of the identification of a minimum approach speed for an antimatter asteroid that allows an impact with Earth.

In the case of matter asteroids, an important parameter is the kinetic energy at impact, which is connected to the amount of damage caused by the asteroid itself. In the case of antimatter asteroids it is instead more important to know the kinetic energy far from Earth, ideally when entering the Solar System, because, being the interaction repulsive, the kinetic energy is going to decrease while getting closer to the Sun, so a low initial kinetic energy could mean not being able to reach Earth orbit at all. At impact, on the contrary, the energy released depends mostly on the annihilation between matter and antimatter, so the velocity is less significant (at least in a first approximation).

To have a first idea of the minimum approach speed we can consider again the two-body problem (since the gravity of the Sun is much bigger than the gravity of other planets) and look at the conservation of energy. Calling "0" the initial state (far from Earth) and "f" the final state (impact with Earth), we can write:

$$\frac{v_0^2}{2} + \frac{\mu}{r_0} = \frac{v_f^2}{2} + \frac{\mu}{r_f} \tag{15}$$

$$v_0 = \sqrt{2\left[\frac{v_f^2}{2} + \mu\left(\frac{1}{r_f} - \frac{1}{r_0}\right)\right]} \tag{16}$$

From these equations it is easy to understand that the minimum values of v_0 can be found for the limit case
$v_f = 0$, so when all kinetic energy has been "consumed" and the asteroid reaches Earth orbit with zero velocity.

Considering a mean value for the radius of Earth orbit, we obtain a minimum velocity (at 60 Astronomical Units from the center of mass of the Sun) of 41.77 km/s. A big value for a man-made spacecraft, but not unrealistic for an astronomical object.

In *Figure 10*, we can see a graph showing the minimum velocity needed to impact Earth versus the distance from the Sun at which it is calculated. The graph has an asymptote at 42.12 km/s, which represents the V_∞, the minimum velocity of the asteroid when the distance tends to infinity.

All of this is valid for the two-body case. In reality we have all the planets of the Solar System, including of course Earth, so the trajectories, and hence the minimum velocities, could be slightly different. To have an idea of how much things change with the complete model, we can use our backward propagating model and put the velocity at Earth to zero, cycling only through time, which happens to be the only variable left. It is possible to see the result in *Figure 11*.

As we can see, the minimum velocity has almost periodical variations through time, due to the interactions with the other planets and to the variations in the radius of Earth's orbit throughout the year. But the presence of the planets can also imply that the minimum velocity can be found with impact velocity different than zero. Since the gravitational effect of the planets compared to that of the Sun is almost negligible, we

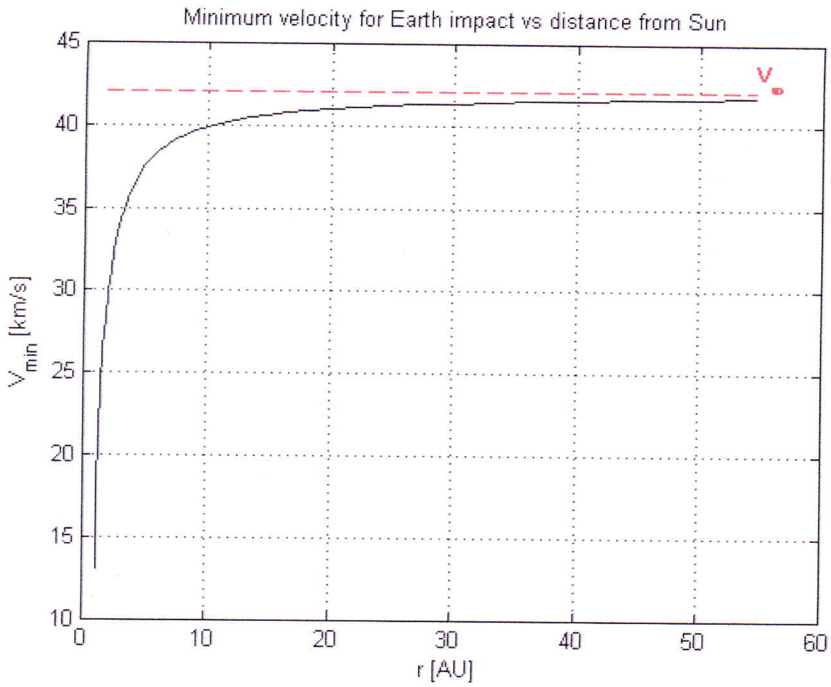

Figure 10 - Minimum velocity needed by an antimatter asteroid to reach Earth calculated at various distances from the Sun using the equations of the 2-body problem and the mean radius of Earth's orbit

do not expect the result to be much different from the two-body case, but there can still be some variations. To verify this hypothesis , we have to check the true trajectories in space that have an impact with Earth to find those with minimum velocity at a given distance from the Sun.

Since a grid search varying all the parameters would be too long and computationally expensive, the best solution is to make an optimization process to find the best trajectory. The three components of velocity at impact and the time of arrival have been used as free parameters, and the modulus of speed has been set at a chosen distance from the Sun as the objective function to be minimized. As a constraint, the velocity has been set to be positive.
The best solution found can be seen in *Figure 12.*

As it is possible to see, the difference from the two-body case is very small, but the solution found is actually better, in terms of low velocity. Many cycles of optimizations have been done varying the initial conditions, since this numerical process is very sensitive to local minima and there is the possibility of missing good solutions because of a wrong initial guess.

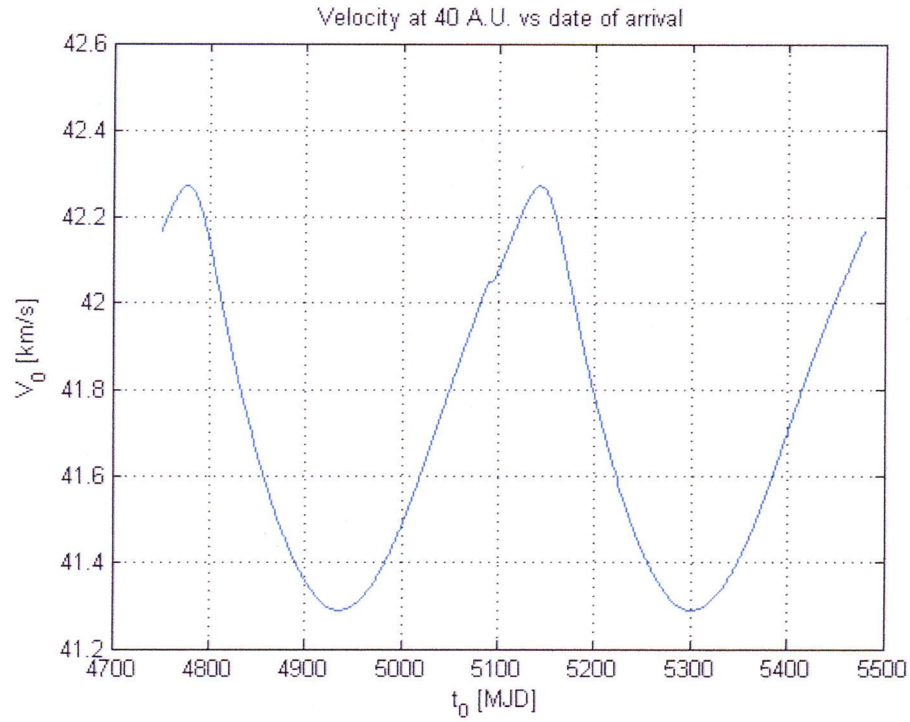

Figure 11 - Minimum velocity needed by an antimatter asteroid to reach Earth at a distance of 40 AU from the Sun, calculated with the integration of dynamical equations of all the major planets, setting final velocity to zero and cycling through time. The time is shown in MJD 2000 (Modified Julian Date, calculated in days starting from 12:00 UTC on 1 Janaury 2000)

Another aspect that can be interesting to investigate is the possible direction of arrival of the asteroid. As we can see, most of figures *Figure* 5 to Figure 9 show trajectories where the impacting antimatter asteroid enters the Solar System in a direction very different from the direction of the Sun. Obviously this depends on the big repulsive action of the Sun itself, that deflects the asteroids too aligned with the Sun-Earth direction away from our planet. We want to verify whether there is a limit in the Sun-Earth-asteroid angle (see *Figure 13*), in order to have collision with Earth. This information could be useful in the future to know where to look more carefully for a possible incoming antimatter asteroid.

Another optimization process has been implemented, using the alpha angle as the objective function to be minimized, and again the three components of velocity at impact and the time of arrival as optimization parameters.

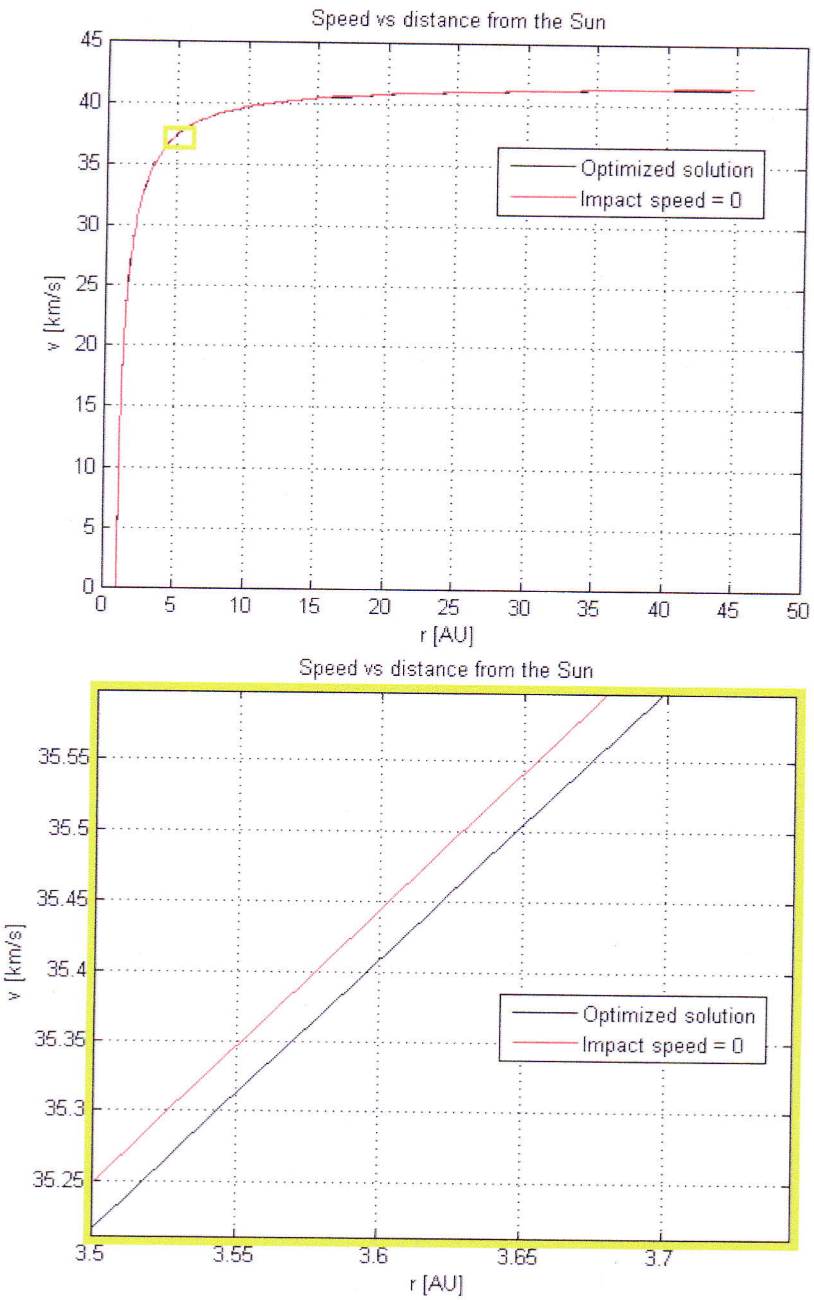

Figure 12 - Best solution found after many cycles of optimizations for the minimum velocity needed by an antimatter asteroid to reach Earth calculated at various distances from the Sun using the complete model. The graph below is a magnification of the graph above in the indicated zone. The red solution is that calculated at the same time of arrival setting the impact speed to zero

Setting no limit to the velocity far from Earth, we obtain a trajectory with a very narrow angle and almost in the shape of a straight line starting very far from the Sun. This is due to the very high velocity of the asteroid (more than 1000 km/s in the example of *Figure 14*), that makes the repulsive effect of the Sun negligible.

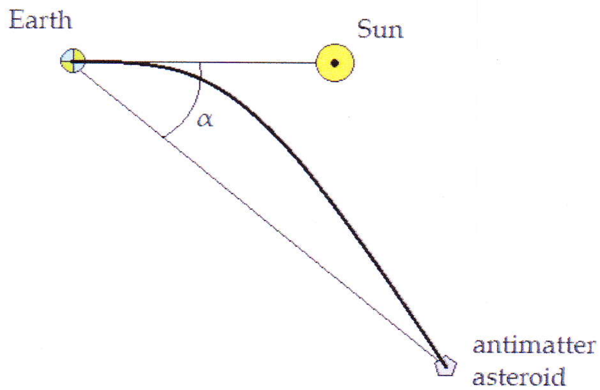

Figure 13 - Schematics of the approach of an antimatter asteroid to our planet, with the indication of the alpha angle between the Earth-Sun vector and the Earth-asteroid vector (using the position of the asteroid at 10 AU from the Sun)

This is kind of a trivial solution, but we can obtain more interesting information by setting a limit to the speed of the asteroid at a given distance from the Sun. By varying this limit it is possible to obtain different values (see *Figure 15*), that represent the minimum alpha angle achievable to impact Earth without exceeding a given velocity at a fixed distance from the Sun (an example is shown in *Figure 16*).

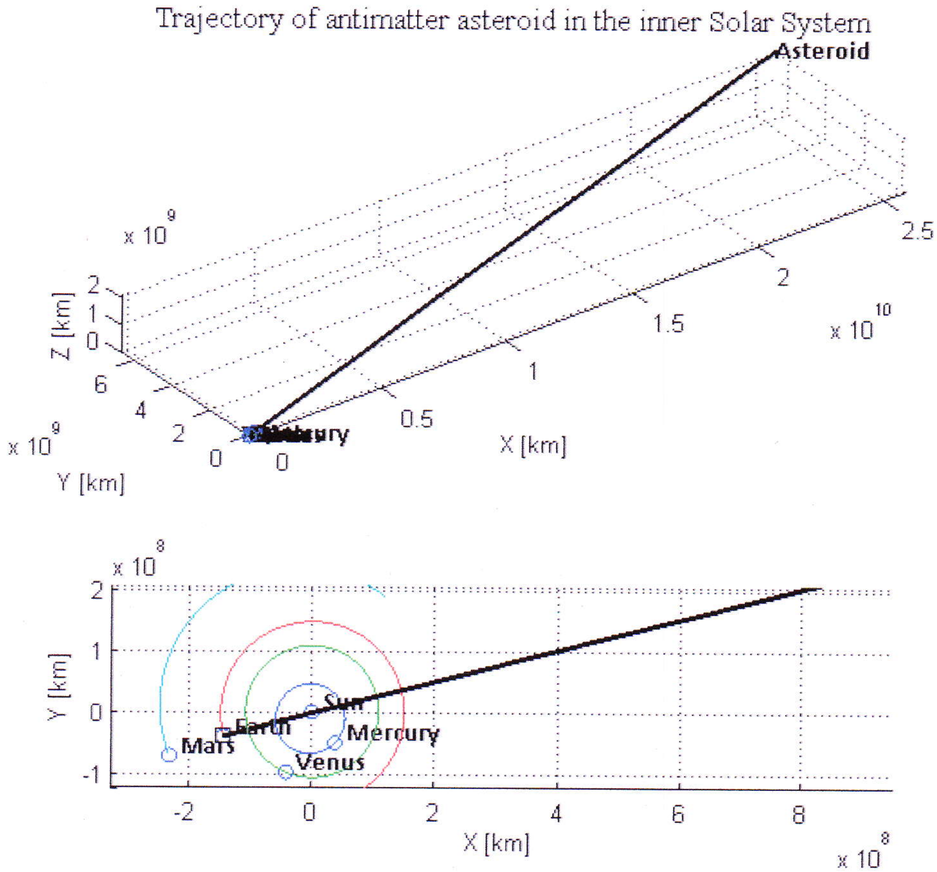

Figure 14 - Trajectory of an antimatter asteroid impacting Earth with as small an alpha angle as possible, calculated with an optimization without constraints on the velocity. In this case the angle obtained is 4.43°, but with a velocity (at 10 AU) of 1037.34 km/sec

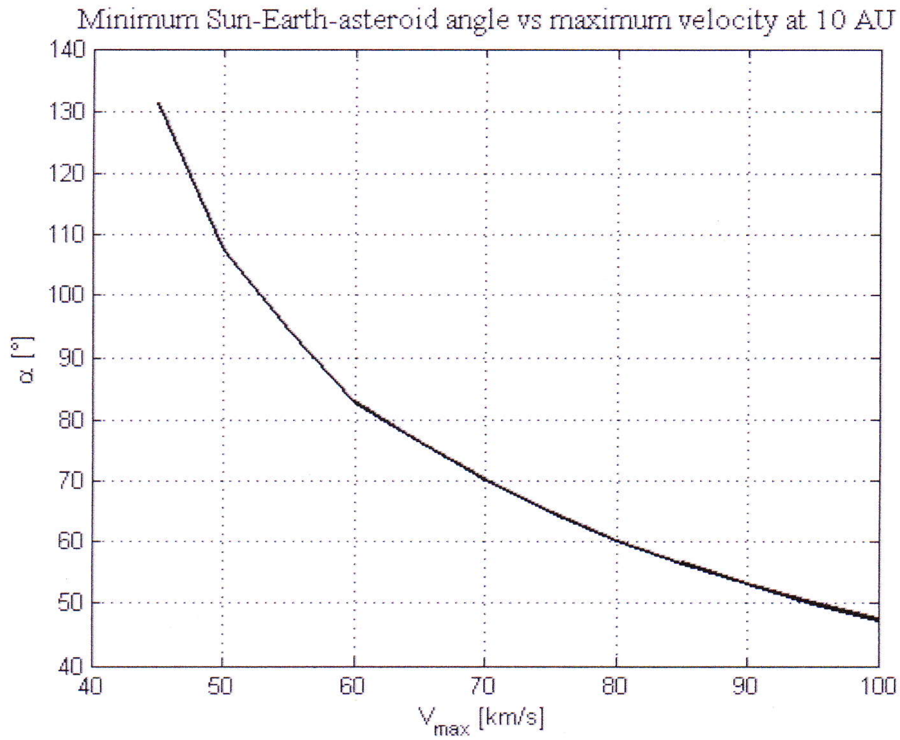

Figure 15 - Minimum alpha angle achievable at different speeds (calculated at 10 AU). This graph has been obtained numerically with different cycles of optimizations, so the values may not belong to the same trajectories, but express a global trend

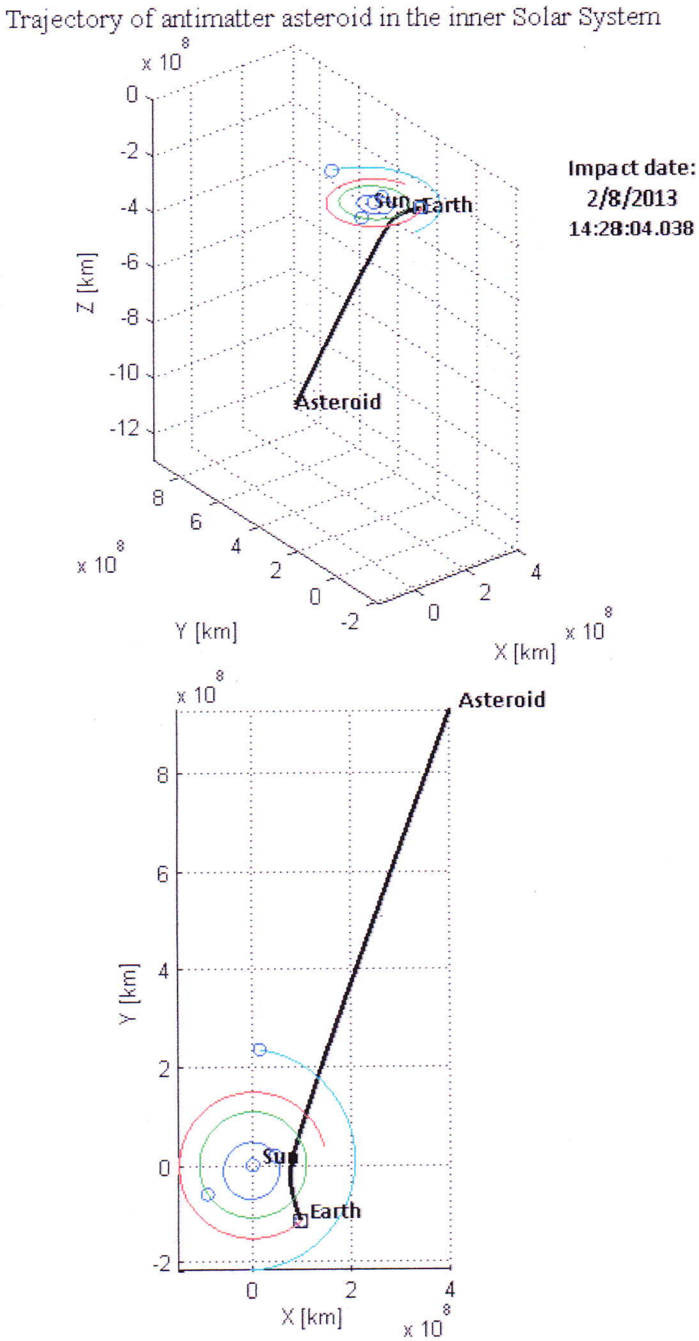

Figure 16 - Example of trajectory with a low alpha angle. Alpha is 69.17°, while the velocity at 10 AU has been limited to 70 km/sec

5. Conclusions

In this paper the author has presented an overview about the problems connected with the trajectories of antimatter bodies entering the Solar System and with the possibilities of impact with Earth. The calculations have been based on Santilli's isodual theory of antimatter [1] because it is the only theory known to the author that has been specifically constructed for the classical representation of neutral antimatter bodies, as needed for antimatter asteroids. In particular, the conjugation from classical and neutral bodies to their antimatter counterparts implies the gravitational repulsion between matter and antimatter which has been assumed at the foundation of these calculations.

The main results obtained can be summarized as follows:

1) An antimatter asteroid can actually enter our Solar System and impact Earth, with initial speeds comparable to those of common celestial bodies

2) An antimatter asteroid would always move around a matter star on an open hyperbolic orbit, with the star itself in the focus opposite to the trajectory.

3) There is a minimum approach speed (roughly 42 km/s) that changes slightly during the year (see *Figure 11*), below which the asteroid would be unable to reach Earth because of the repulsive action of the Sun.

4) Depending on the velocity of the asteroid (see *Figure 15*), there is a minimum angle between the approaching direction of the asteroid and the Sun (as seen from the Earth), below which impact with Earth is not possible.

Of course the model used to achieve these results is not complete, and the results themselves are valid under the correct hypotheses: point-like masses moving in vacuum, asteroid mass negligible with respect to the other masses involved, no interaction other than gravity, so forces are conservative and motion is reversible. These conditions approximate well the phenomena considered in the real world, but there are some effects that have not been considered.

The interaction with solar light has been ignored, in part because its entity is supposed to be secondary with respect to the gravitation, considering also the high speeds involved, and in part because the effect of matter light hitting an antimatter body is still unknown (see [4]).

The effect of the solar wind hitting the asteroid has also been ignored. The solar wind is mainly composed of protons and electrons escaping the Sun's gravity, so at their

impact with the asteroid they would annihilate releasing energy that would probably modify the asteroid's trajectory during time. The entity of this effect would depend mostly on the distance from the Sun and the density and speed of the solar wind itself, which is very variable during time.
Another effect ignored in this study is that of the asteroid belt, located between Mars and Jupiter orbits, that again could slightly modify the trajectory, but this effect is supposed to be very small if compared to the gravitation of the Sun and other planets.

Further studies are needed to better understand the behavior of these bodies, since the research in this field is only at the beginning. Future developments should focus on:

- the effect of matter light on antimatter, not only for the consequences on the trajectories, but also for the detection itself of the antimatter asteroids (as explained in [4]) without which we wouldn't be able to locate an incoming asteroid. Important steps forward have been made for the detection of antimatter stars and galaxies (see [2] and [3]), but we still don't know anything about cold bodies that don't have emissions of their own;

- the effect of the solar wind on the trajectories of antimatter bodies. Although weak, especially far from the Sun, for an approaching asteroid this effect could become more significant with the passage of time. Moreover, the annihilation of electrons and protons with the asteroid's surface would produce both matter and antimatter radiations, and the former, if intense enough, could be seen from Earth and contribute to the detection of the asteroid itself, so this aspect could be particularly important to investigate on.

- the effects of the entrance into the atmosphere, that would be important of course to determine the behavior of the antimatter body and the damages it could deliver to our planet.

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